Durham District School Board

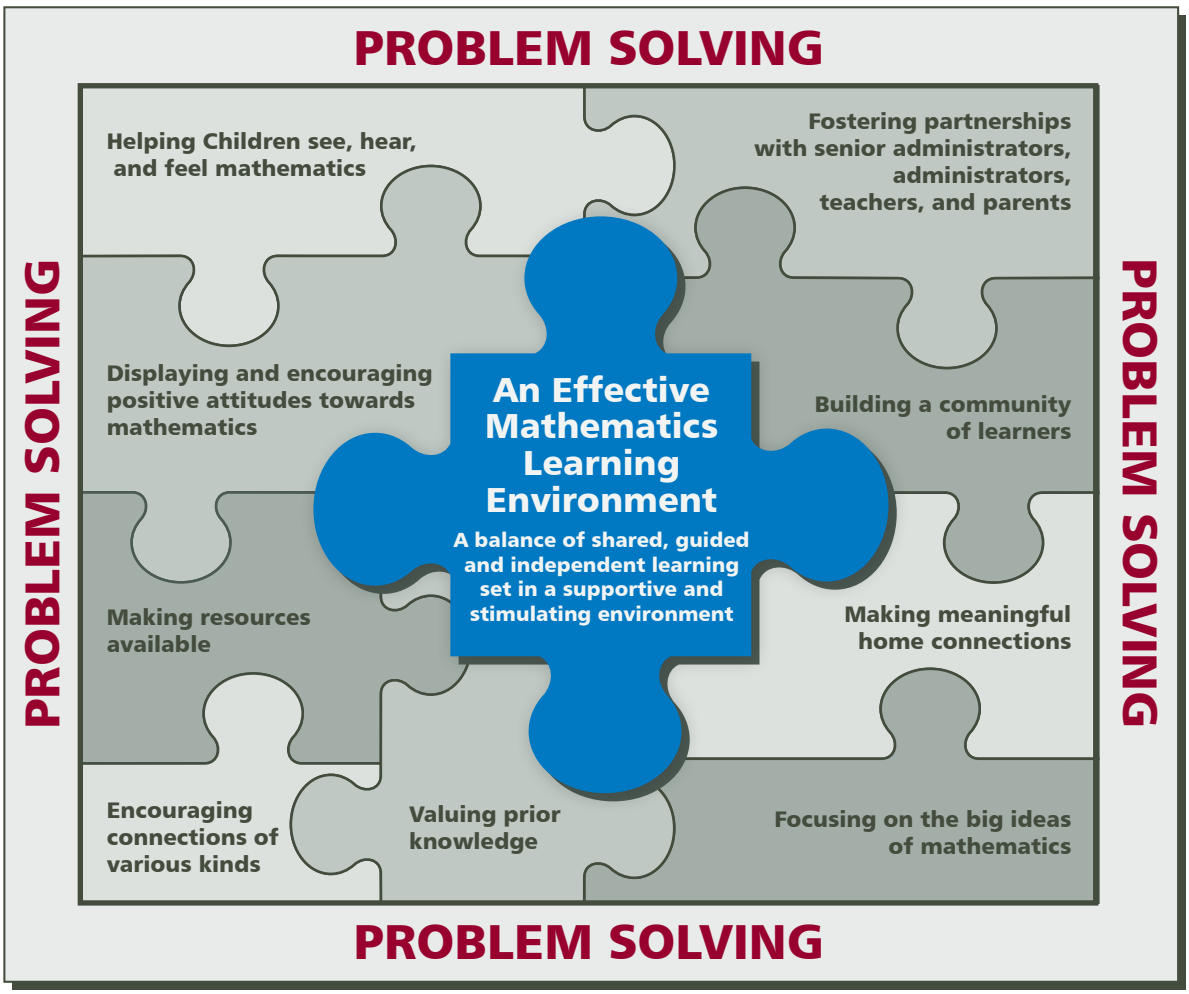
Mathematical Profile

A COMPREHENSIVE APPROACH

Balance procedural understanding and conceptual understanding.
Balance whole group and small group instruction.
Balance teacher directed and student-focused learning.

KEY MESSAGE 1: Effective mathematics learning environments are challenging, developmentally appropriate for all students, and strategically organized.

KEY MESSAGE 2: Effective mathematics programs include problem-solving-based mathematical tasks and lessons that engage students at their level of readiness and provoke them to develop coneptual understanding, to generate, select and use appropriate procedures/algorithms, problem-solving strategies and to activate multiple representations of their mathematical thinking.



Early Math Strategy: the Report of the Expert Panel on Early Math in Ontario, Ministry of Education 2003, pg. 12.

KEY MESSAGE 3: Effective mathematics instruction engages all students in developing deep conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, clear and precise mathematical communication, and a positive and productive disposition towards mathematics.

KEY MESSAGE 4: Effective mathematics assessment is on-going and promotes continual growth in students' mathematics learning over time.

KEY MESSAGE 5: Teachers need time and support to develop "pedagogical content knowledge"; that is, the knowledge of mathematics for teaching.

CATEGORIES	ACHIEVEMENT CHART FOR GRADES 1-12
KNOWLEDGE AND UNDERSTANDING Subject specific content acquired in each grade (knowledge), and the comprehension of its meaning and significance (understanding).	**Knowledge of content** (e.g., facts, terms, procedural skills, use of tools) **Understanding of mathematical concepts**
THINKING The use of critical and creative thinking skills and/or processes.	**Use of planning skills** • understand the problem (e.g., formulating and interpreting the problem, making conjectures) • making a plan for solving **Use of processing skills** • carrying out a plan (e.g., collecting data, questioning, testing, revising, modeling, solving, inferring, forming conclusions) • looking back at the solution (e.g., evaluating reasonableness, making convincing arguments, reasoning, justifying, proving, reflecting) **Use of critical/creative thinking processes** (e.g., problem solving, inquiry)
COMMUNICATION The conveying of meaning through various forms	**Expression and organization of ideas and mathematical thinking** (e.g., clarity of expression, logical organization), **using oral, visual, and written forms** (e.g., pictorial, graphic, dynamic, numeric, algebraic forms; concrete materials) **Communication for different audiences** (e.g., peers, teachers) **and purposes** (e.g., to present data, justify a solution, express a mathematical argument) **in oral, visual, and written forms** **Use of conventions, vocabulary, and terminology of the discipline** (e.g., terms, symbols) **in oral, visual, and written forms**
APPLICATION The use of knowledge and skills to make connections within and between various contexts	**Application of knowledge and skills in familiar contexts** **Transfer of knowledge and skills to new contexts** **Making connections within and between various contexts** (e.g., connections between concepts, representations, and forms within mathematics; connections involving the use of prior knowledge and experience; connections between mathematics, other disciplines , and the real world)

UNDERSTAND THE PROBLEM
(the exploratory stage)

- reread and restate the problem
- identify the information given and the information that need to be determined

Communication: talk about the problem to understand it better

MAKE A PLAN

- relate the problem to similar problems solved in the past
- consider possible strategies
- select a strategy or a combination of strategies

Communication: discuss ideas with others to clarify which strategy or strategies would work best

CARRY OUT THE PLAN

- execute the chosen strategy
- do the necessary calculations
- monitor success
- revise or apply different strategies as necessary

Communication:

- draw pictures; use manipulatives to represent interim results
- use words and symbols to represent the steps in carrying out the plan or doing the calculations
- share results of computer or calculator operations

LOOK BACK AT THE SOLUTION

- check the reasonableness of the answer
- review the method used: Did it make sense? Is there a better way to approach the problem?
- consider extensions or variations

Communication: describe how the solution was reached, using the most suitable format, and explain the solution.

ACHIEVEMENT CHART CATEGORIES FOR MATHEMATICS GRADE 1-12

This chart identifies four categories of achievement that are interrelated, reflecting the wholeness and interconnectedness of learning. Teachers use this chart to make judgments about the conversations, observations and products of student learning based on a body of evidence collected over time (Ontario Math Curriculum, 2005, Revised).

This model is inherent in the "thinking category" of the achievement chart.

K-2 teachers can use this model to guide questioning and prompting during problem solving (not taught directly)

By grade 3, teachers can present the model more explicitly and incorporate connections from earlier grades.

3 PART LESSON PLAN

TEACHING THROUGH PROBLEM SOLVING AND BIG IDEAS

WHAT ARE BIG IDEAS?

"Big ideas are the enduring understandings that underpin the K-12 mathematics curricula…they are similar to the Mathematical Processes in that they are fundamental content concepts that repeatedly emerge and grow in the study of mathematics" (Math Gains).

WHY FOCUS ON BIG IDEAS?

"A focus on Big Ideas assists students in making connections between one thing they learn in mathematics and another. It is the repeated exposure to and discussion of a Big Idea that helps students see its value. Thinking about Big Ideas also assists the teacher in building connections into lesson planning" (Math Gains).

(Ontario, Ministry of Education. Math Gains. Assessment for Learning Video Series: Learning Goals and Success Criteria Viewing Guide. p. 7)

1

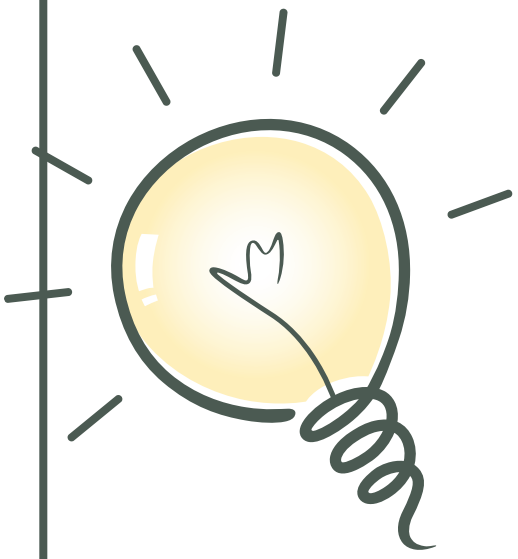
GETTING STARTED/ MINDS ON

Describe how you will introduce the learning activity to your students. What key questions will you ask? How will you gather diagnostic or formative data about the students' current levels of understanding? How will students be grouped? How will material be distributed?

2

WORKING ON IT/ ACTION

Describe the task in which your students will be engaged. What misconceptions or difficulties do you think they might experience? How will they demonstrate their understanding of the concept? How will you gather your assessment data (e.g. checklist, anecdotal records)? What extension activities will you provide?



3

CONSOLIDATION

How will you clarify explicit learning goals and success criteria so that the students can reflect on them? What key questions will you ask during the debriefing? Exit ticket design.

HIGH TEACHER SUPPORT

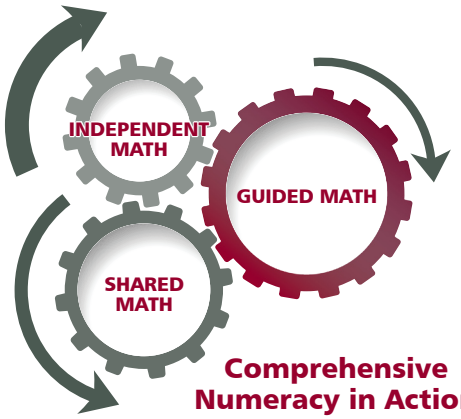
GRADUAL RELEASE OF RESPONSIBILITY

LOW TEACHER SUPPORT

SHARED MATH

TEACHERS COULD BE:

- facilitating, observing and asking key questions as students work
- promoting individual, small group or whole group discussion
- reading mathematics-related literature/ stories
- linking mathematics to authentic situations through rich tasks
- gathering assessment data through the use of targeted diagnostic tools such as Leaps and Bounds, PRIME



Comprehensive Numeracy in Action

STUDENTS COULD BE:

- working in partners to explore mathematical concepts through the four-step problem solving model (grade 3 and up)
- working at centres in small groups to explore concepts, make connections to other math concepts and ask questions
- working with peers to support each other's math thinking
- using manipulatives to explore math concepts
    - engaged in learning through a game, puzzle, software, songs, literature and other dynamic activities to support the math learning
    - exploring concepts as a way to deepen and explore new mathematical thinking through reasoning and proving
    - talking and sharing their mathematical ideas whole group through Bansho or Math Congress

WHAT I MIGHT SEE?

- learning goals/success criteria support the conceptual understanding, are co-constructed and scaffolded into the learning
- students are engaged in math discussions using specific math vocabulary
- shared math may occur between teacher and whole class , teacher and small group or student to student
- reflection, discussion and sharing occur at the end of the lesson to bring closure and clarification to the key mathematical ideas
- grouping could be small groups or whole class

GUIDED MATH

TEACHERS COULD BE:

- activating the concept and connecting it with prior knowledge
- modeling mathematical language, problem solving and thinking
- facilitating the discussion and sharing effective questions
- setting up a learning experience so that students gain new skills
- highlighting a variety of strategies through the big ideas
- clarifying new knowledge developmentally

STUDENTS COULD BE:

- responding to teacher's questions and offering next steps
- guiding and modeling mathematical thinking or ideas for peers with teacher guidance
- engaging in accountable talk related to the mathematical concepts/big ideas of the lesson
- developing their communication skills in math (orally and in writing) to understand the concepts with more depth

WHAT I MIGHT SEE?

- focused lessons
- instruction is sequential and planned by the teacher
- class instruction is planned, flexible, and purposeful and yet capitalizes on alternative ideas and strategies
- the teacher works with small group, and at times, with individual students
- reflection, discussion and sharing utilizing manipulatives, charts/displays, whiteboard, or sitting on the floor.

INDEPENDENT MATH

TEACHERS COULD BE:

- facilitating, observing and asking key questions as students work
- observe the students as they demonstrate their understanding, practice a skill or consolidate learning in a developmentally appropriate manner through independent work

STUDENTS COULD BE:

- demonstrating their understanding, practicing a new skill
- grappling with a problem on their own and selecting tools and strategies to support their attempts
- consolidating ideas for and by themselves
- using manipulatives
- playing games
- participating in a math walk
- using technology at the point of learning to support the math concepts
- completing an exit ticket from a three-part math lesson

WHAT I MIGHT SEE?

- may occur at various times and not just at the end of the activity or lesson
- may include practicing a mathematical skill, journal writing, explaining an idea to the teacher, playing an independent game, using technology, or using manipulatives to gain a better grasp of a key concept(s)
- students working on learning goals and utilizing co-constructed, student-friend success criteria
- 1:1 conference with teacher on key concept(s)

IMPROVING COMMUNICATION IN MATHEMATICS CLASSROOMS

Ways to co-ordinate whole class discussions

GALLERY WALK

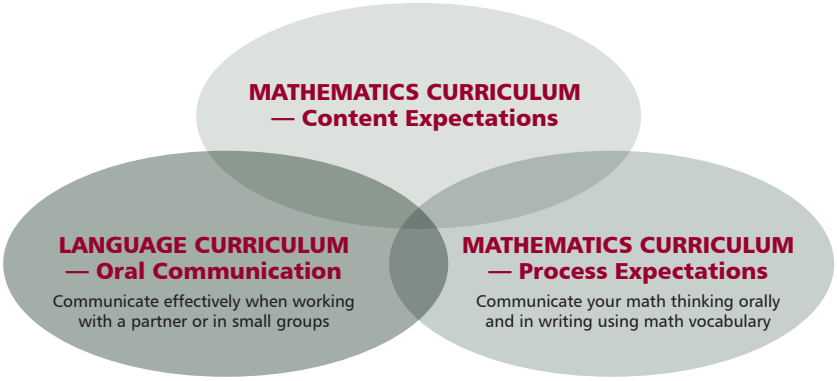
- In a gallery walk, students have an opportunity to discuss in small groups different solutions and strategies.
- It provides students with a chance to give and receive descriptive feedback on their mathematics from peers and the teacher.
- Two common forms of a gallery walk include:
  - *Students travelling with their math partners to various solutions in the classroom and discussing the math and recording feedback for groups on sticky notes.*
  - *A stay and stray method—if working in larger groups (3 or 4) you can have one member stay to talk about the math while the other members visit other groups to discuss various solutions.*
- As students are participating in a gallery walk, the teacher can take time to listen for math vocabulary, make observations of misconceptions or strategies that can then be highlighted in the class discussion.
- Once students have had a chance to visit other solutions, they come back and reflect on their solution and make any necessary adjustments before the class discussion.
- The teacher will then lead a class discussion of solutions and strategies.

MATH CONGRESS

- A math congress focuses on two or three solutions in an effort to develop mathematical thinking and learning.
- The teacher must strategically select the solutions to discuss—the focus is on reasoning and communicating mathematical thinking about a few big ideas and takes into account that all students are not at the same place mathematically.
- The goal is to scaffold the learning—using student strategies to further the understanding.
- Students take time to prepare for the congress— organizing their information to present and preparing their arguments.
- The teacher starts the discussion but the entire class is part of a congress, talking about the strategy, questioning the presenters, asking questions like 'will that always work?' 'how are the strategies similar and different?'
- This type of consolidation takes some work and training in the classroom—students need to be part of a community of mathematicians, comfortable to share ideas and question.

BANSHO

- Bansho is Japanese for 'Board Writing'.
- It is literally a written expression of the mathematical process of a lesson.
- All information is recorded on the board, including all three parts of the lesson—and it remains on the board as students discuss the various solutions.
- Again the teacher carefully selects which solutions will be placed on the board (2 to 4 solutions usually).
- The solutions chosen should help students in the discussion to develop their understanding of the learning goal for the lesson—criteria can include the mathematical strategy, relationships between concepts, standard and alternative algorithms, etc.
- Solutions are not organized by levels of achievement— usually organization would start with solutions that show some conceptual knowledge, moving towards more efficient strategies and algorithms and finally to solutions that show generalizations between concepts and idea.
- Teachers can ask themselves some questions to determine how they would organize the solutions:
  - *What math approaches are the students using to solve the problem? Does it relate to the learning goal?*
  - *Which solutions are conceptual? Which are more efficient? Which have the potential to be generalized to the big idea?*
  - *How are the solutions related? To each other? To the learning goals of the lesson?*



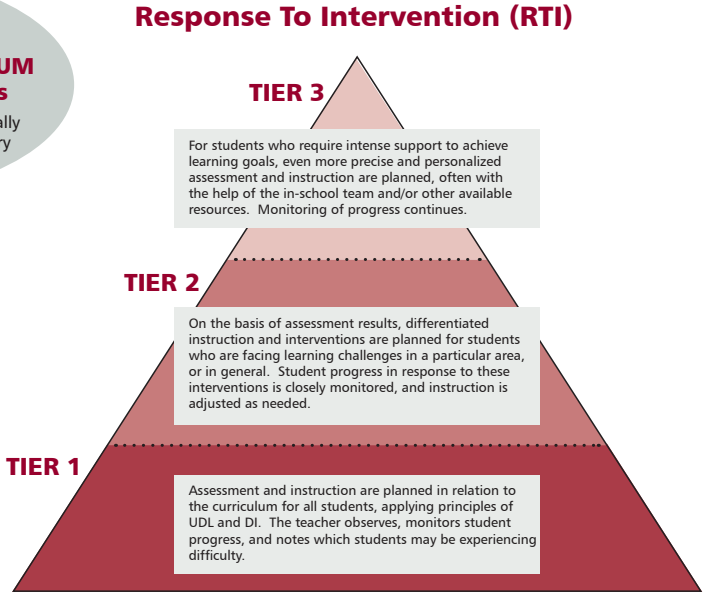


Figure 5. The Tiered Approach From Learning for ALL 2011, page 22.

LEAPS AND BOUNDS **PRIME** **GAP CLOSING**

Leaps and Bounds, **J/I Gap Closing** and **I/S Gap Closing** were developed to help teachers support students who are struggling in mathematics. It supports students working as many as 3 levels below grade. Diagnostic assessments for every topic precisely pinpoint significant gaps in students' understanding. You will find strategic lessons and questions for DI that enable teachers to build on students' knowledge and those critical gaps in understanding. **PRIME** is a diagnostic tool, which assesses conceptual understanding in each strand, using a developmental phases map. Educators use the Developmental Maps to identify a student's phase of mathematics understanding, pin-pointing exactly what each student can do within that phase.

SMALL GROUP INTERVENTION

- Teacher Moderation
- On Going Assessment and Feedback
- Three-part Problem Solving Based Lesson
- Use of Learning Materials Appropriate to the Mathematics

THE SEVEN MATHEMATICAL PROCESSES

The teacher should ensure that the seven mathematical processes are embedded throughout student learning.

PROBLEM SOLVING

Problem solving is central to learning mathematics. By learning to solve problems and by learning through problem solving, students are given numerous opportunities to connect mathematical ideas and to develop conceptual understanding.

REASONING AND PROVING

The reasoning process supports a deeper understanding of mathematics by enabling students to make sense of the mathematics they are learning. The process involves exploring phenomena, developing ideas, making mathematical conjectures, and justifying results.

REFLECTING

Good problem solvers regularly and consciously reflect on and monitor their own thought process. By doing so, they are able to recognize when the technique they are using is not effective, and to make a conscious decision to switch to a different strategy, rethink the problem and so forth.

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES

Students need to develop the ability to select the appropriate electronic tools, manipulatives, and computational strategies to perform particular mathematical tasks, to investigate mathematical ideas, and to solve problems.

CONNECTING

Experiences that allow students to make connections (to see, for example, how concepts and skills from one strand of mathematics are related to those from another) will help them to grasp general mathematical principles.

REPRESENTING

In elementary school mathematics, students represent mathematical ideas and relationships and model situations using concrete materials, pictures, diagrams, graphs, tables, numbers, words and symbols. Learning the various forms of representation helps students to understand mathematical concepts and relationships; communicate their thinking, arguments, and understandings; recognize connections among related mathematical concepts; and use mathematics to model and interpret realistic problem situations.

COMMUNICATING

Communication is the process of expressing mathematical ideas. It involves understanding orally, visually, and in writing, using numbers, symbols, pictures, graphs diagrams, and words.

EFFECTIVE QUESTIONING
Questions And Prompts To Get Students Thinking

TO HELP STUDENTS SHARE THEIR REPRESENTATIONS
(and show / describe / demonstrate / represent)

Questions to pose:
- How have you shown your thinking (e.g., picture, model, number, sentence)?
- Which way (e.g., picture, model, number, sentence) best shows what you know?
- How have you used math words to describe your experience?
- How did you show it?
- How would you explain _____ to a student in Grade __? (a grade lower than the one the student is in)

Prompts to use:
- I decided to use a ...
- A graph (table, T-chart, picture,) shows this the best because...
- I could make this clearer by using a...
- The math words that help someone understand what I did are...

TO HELP STUDENTS REFLECT ON THEIR WORK
(and analyze / compare / contrast / test / survey / classify / sort/ use / apply / model)

Questions to pose:
- What mathematics were you investigating?
- What questions arose as you worked?
- What were you thinking when you made decisions or selected strategies to solve the problem?
- What changes did you have to make to solve the problem?
- What was the most challenging part of the task? And why?
- How do you know?
- How does knowing_____help you to answer the questions _____?

Prompts to use:
- A question I had was ...
- I was feeling really ...
- I decided to _____, I was thinking ...
- I found _____ challenging because ...
- The most important thing I learned in math today is ...

TO HELP STUDENTS MAKE CONNECTIONS
(and connect / relate / refer / imagine / describe / compare)

Questions to pose:
- What does this make you think of?
- What other math can you connect with this ?
- When do you use this math at home? At school? In other places?
- Where do you see _____ at school? At home? Outside?
- How is this like something you have done before?

Prompts to use:
- This new math idea is like...
- I thought of ...
- I did something like this before when ...
- We do this at home when we ...
- I remember when we ...

TO HELP STUDENTS SHARE THEIR FEELINGS, ATTITUDES OR BELIEFS ABOUT MATHEMATICS
(and share / reflect / describe / compare / tell)

Questions to pose:
- What else would you like to find out about _____?
- How do you feel about mathematics?
- How do you feel about _____?
- What does the math remind you of?
- How can you describe math?

Prompts to use:
- The thing I like best about mathematics is ...
- The hardest part of this unit on _____ is ...
- I need help with _____ because ...
- Write to tell a friend how you feel about what we are doing in mathematics.
- Mathematics is like _____ because ...
- Today, I felt ...

TO HELP STUDENTS RETELL
(and tell / list / recite / name / find / describe / explain / illustrate / summarize)

Questions to pose:
- How did you solve the problem?
- What did you do?
- What strategy did you use?
- What math words did you use or learn?
- What were the steps involved?
- What did you learn today?
- What do(es) _____ mean to you?

Prompts to use:
- I solved the problem by ...
- The math words I used were ...
- The steps I followed were ...
- My strategy was successful because ...
- Explain to a young child or someone that wasn't involved ...
- Draw a picture to show how you solved the problem.

TO HELP STUDENTS PREDICT, INVENT OR PROBLEM SOLVE
(and create / plan / design / predict / imagine / devise / decide / defend / solve / debate)

Questions to pose:
- What would happen if ... ?
- What decisions can you make from the patterns that you discovered?
- How else might you have solved the problem?
- Will it be the same if we use different numbers?
- What things in the classroom have these same shapes?
- How is this pattern like addition?
- What would you measure it with? Why ?
- How are adding and multiplying the same?

Prompts to use:
- Prove that there is only one possible answer to this problem.
- Convince me!
- Tell me what is the same? What is different?
- How do you know?

MINISTRY RESOURCES
to support comprehensive numeracy

Guides to Effective Instruction K-6: (instructional guides)

Volume One. Foundations of mathematics instruction:
http://www.eworkshop.on.ca/edu/resources/guides/Guide_Math_K_6_Volume_1.pdf

Volume Two. Problem solving and communication:
http://www.eworkshop.on.ca/edu/resources/guides/Guide_Math_K_6_Volume_2.pdf

Volume Three. Classroom resources and management:
http://www.eworkshop.on.ca/edu/resources/guides/Guide_Math_K_6_Volume_3.pdf

Volume Four. Assessment and home connections:
http://www.eworkshop.on.ca/edu/resources/guides/Guide_Math_K_6_Volume_4.pdf

Volume Five. Teaching basic facts and multidigit computations:
http://www.eworkshop.on.ca/edu/resources/guides/Guide_Math_K_6_Volume_5.pdf

Primary Guides to Effective Instruction in Math (Strand Specific):

Number Sense and Numeration:
http://www.eworkshop.on.ca/edu/resources/guides/Guide_Math_K_3_NSN.pdf

Geometry and Spatial Sense:
http://www.eworkshop.on.ca/edu/resources/guides/Guide_Math_K_3_GSS.pdf

Data Management and Probability:
http://www.eworkshop.on.ca/edu/resources/guides/Data_Management_and_Probability_K-3.pdf

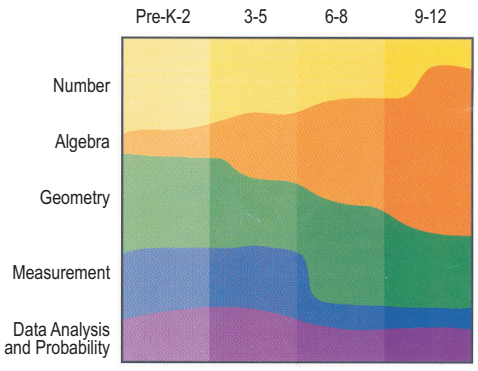
Measurement:
http://www.eworkshop.on.ca/edu/resources/guides/Measurement_K-3.pdf

Patterning and Algebra:
http://www.eworkshop.on.ca/edu/resources/guides/Patterning_and_Algebra_K-3.pdf

General Math Support:

2005 Ontario Curriculum, Grades 1–8: Mathematics (Rev. ed.):
http://www.edu.gov.on.ca/eng/curriculum/elementary/math.html

RELATIVE EMPHASIS OF STRANDS BY DIVISION



Online teaching resources at:
http://www.eworkshop.on.ca

Ministry of Education of Ontario. Math GAINS resources:
http://www.edugains.ca/newsite/gainspop/mathpop.html|

Literacy and Numeracy Secretariat. Webcasts for Educators:
http://www.curriculum.org/secretariat/literacy_en.shtml

www.eqao.com

EQAO's Understanding Levels of Achievement
http://www.eqao.com/pdf_e/12/UnderstandingLevelsAchievemet_PrimaryDivision_en.pdf and http://www.eqao.com/pdf_e/12/UnderstandingLevelsAchievemet_JuniorDivision_en.pdf

Junior: Strand Specific Math Guides to 4-6

Data Management:
http://www.eworkshop.on.ca/edu/resources/guides/Guide_Data_Management_Probability_456.pdf

Geometry :
http://www.eworkshop.on.ca/edu/resources/guides/Guide_Geometry_Spatial_Sense_456.pdf

Measurement:
http://www.eworkshop.on.ca/edu/resources/guides/Guide_Measurement_456.pdf

Patterning: http://www.eworkshop.on.ca/edu/resources/guides/Guide_Patterning_and_Algebra_456.pdf

Junior Guides to Effective Instruction in Math: (Number Sense and Numeration)

Volume One. The big ideas:
http://www.eworkshop.on.ca/edu/resources/guides/NSN_vol_1_Big_Ideas.pdf

Volume Two. Addition and subtraction:
http://www.eworkshop.on.ca/edu/resources/guides/NSN_vol_2_Addition_Subtraction.pdf

Volume Three. Multiplication:
http://www.eworkshop.on.ca/edu/resources/guides/NSN_vol_3_Multiplication.pdf

Volume Four. Division:.
http://www.eworkshop.on.ca/edu/resources/guides/NSN_vol_4_Division.pdf

Volume Five. Fractions
http://www.eworkshop.on.ca/edu/resources/guides/NSN_vol_5_Fractions.pdf

Volume Six. Decimal numbers
http://www.eworkshop.on.ca/edu/resources/guides/NSN_vol_6_Decimal_Numbers.pdf

Manipulatives	JK-3	4-6	7-8	9-12
1 cm Interlocking Cubes (snap cubes)	✓	✓		
2 cm Interlocking Cubes (snap cubes)	✓	✓	✓	✓
Algebra Tiles			✓	✓
Attribute Blocks	✓			
Base 10 Materials	✓	✓	✓	
Coloured Tiles	✓			
Compasses			✓	✓
Cuisinaire Rods	✓	✓	✓	
Numbered Cubes	✓	✓	✓	✓
Geoboards	✓	✓	✓	✓
Hundreds Chart / Carpet / Board	✓	✓		
Metric Rulers	✓	✓	✓	✓
Miras	✓	✓	✓	✓
Money	✓	✓		
Nets	✓	✓	✓	✓
Pattern Blocks	✓	✓	✓	
Playing Cards	✓	✓	✓	✓
Power Polygons		✓	✓	✓
Power Solids		✓	✓	✓
Protractors		✓	✓	✓
3D Models	✓	✓		
Tangrams	✓	✓	✓	
Tape Measures		✓	✓	✓
Two Sided Counters	✓		✓	